

§1. Задача Гурса. Мажорантные оценки

Формулы (13)-(15). Из формулы (13) получаем:

$$z_1 = - \int_0^y \int_0^x \left\{ a \frac{\partial z_0}{\partial \xi} + b \frac{\partial z_0}{\partial \eta} + cz_0 \right\} d\xi d\eta \Rightarrow |z_1| \leq 3MHxy \leq 3MH \frac{(x+y)^2}{2!}$$

$$\frac{\partial z_1}{\partial x} = - \int_0^y \left\{ a \frac{\partial z_0}{\partial x} + b \frac{\partial z_0}{\partial \eta} + cz_0 \right\} d\eta \Rightarrow \left| \frac{\partial z_1}{\partial x} \right| \leq 3MHy \leq 3MH(x+y)$$

Аналогично:

$$\left| \frac{\partial z_1}{\partial y} \right| \leq 3MHx \leq 3MH(x+y)$$

Индукция:

$$z_n = - \int_0^y \int_0^x \left\{ a \frac{\partial z_{n-1}}{\partial \xi} + b \frac{\partial z_{n-1}}{\partial \eta} + cz_{n-1} \right\} d\xi d\eta$$

$$|z_{n-1}| \leq 3HM^{n-1} K^{n-2} \frac{(x+y)^n}{n!}$$

$$\left| \frac{\partial z_{n-1}}{\partial x} \right| \leq 3HM^{n-1} K^{n-2} \frac{(x+y)^{n-1}}{(n-1)!}$$

$$\left| \frac{\partial z_{n-1}}{\partial y} \right| \leq 3HM^{n-1} K^{n-2} \frac{(x+y)^{n-1}}{(n-1)!},$$

где $K = L + 2$.

$$|z_n| \leq 3HM^n K^{n-2} \left\{ \frac{2}{(n-1)!} \int_0^y \int_0^x (\xi + \eta)^{n-1} d\xi d\eta + \frac{1}{n!} \int_0^y \int_0^x (\xi + \eta)^n d\xi d\eta \right\},$$

$$\int_0^y \int_0^x (\xi + \eta)^{n-1} d\xi d\eta \leq \frac{1}{n} \int_0^y (x + \eta)^n d\eta \leq \frac{1}{n(n+1)} (x+y)^{n+1},$$

$$\int_0^y \int_0^x (\xi + \eta)^n d\xi d\eta \leq \frac{1}{(n+1)(n+2)} (x+y)^{n+2},$$

$$\{\dots\} \leq \frac{2}{(n-1)!} \frac{1}{n(n+1)} (x+y)^{n+1} + \frac{1}{n!} \frac{1}{(n+1)(n+2)} (x+y)^{n+2} =$$

$$= \frac{2}{(n+1)!} (x+y)^{n+1} + \frac{1}{(n+2)!} (x+y)^{n+2} = \frac{(x+y)^{n+1}}{(n+1)!} \left(2 + \frac{x+y}{n+2}\right) <$$

$$< \frac{(x+y)^{n+1}}{(n+1)!} \left(2 + \frac{2L}{n+2}\right) < \frac{(x+y)^{n+1}}{(n+1)!} (2+L) = \frac{(x+y)^{n+1}}{(n+1)!} K \Rightarrow$$

$$|z_n| \leq 3HM^n K^{n-1} \frac{(x+y)^{n+1}}{(n+1)!} .$$

$$\frac{\partial z_n}{\partial x} = - \int_0^y \left\{ a \frac{\partial z_{n-1}}{\partial x} + b \frac{\partial z_{n-1}}{\partial \eta} + c z_{n-1} \right\} d\eta ,$$

$$\left| \frac{\partial z_n}{\partial x} \right| \leq 3HM^n K^{n-2} \left\{ \frac{2}{(n-1)!} \int_0^y (x+\eta)^{n-1} d\eta + \frac{1}{n!} \int_0^y (x+\eta)^n d\eta \right\} ,$$

$$\{ \dots \} = \frac{2}{(n-1)!} \frac{(x+y)^n}{n} + \frac{1}{n!} \frac{(x+y)^{n+1}}{n+1} = 2 \frac{(x+y)^n}{n!} + \frac{(x+y)^{n+1}}{(n+1)!} ,$$

$$\left| \frac{\partial z_n}{\partial x} \right| \leq 3HM^n K^{n-2} \frac{(x+y)^n}{n!} \left(2 + \frac{(x+y)}{n+1}\right)^{< 2L} =$$

$$= 3HM^n K^{n-2} \frac{(x+y)^n}{n!} \left(2 + \frac{2}{n+1} L\right)^{< 2+L=K} < 3HM^n K^{n-1} \frac{(x+y)^n}{n!} .$$

Аналогично:

$$\left| \frac{\partial z_n}{\partial y} \right| < 3HM^n K^{n-1} \frac{(x+y)^n}{n!}$$

$$|z_n| \leq 3HM^n K^{n-1} \frac{(x+y)^{n+1}}{(n+1)!} < 3HM^n K^{n-1} \frac{(2L)^{n+1}}{(n+1)!} = \frac{3H}{K^2 M} \frac{(2KLM)^{n+1}}{(n+1)!} ,$$

$$\left| \frac{\partial z_n}{\partial x} \right| \leq 3HM^n K^{n-1} \frac{(x+y)^n}{n!} < 3HM^n K^{n-1} \frac{(2L)^n}{n!} = \frac{3H}{K} \frac{(2KLM)^n}{n!} ,$$

$$\left| \frac{\partial z_n}{\partial y} \right| \leq \frac{3H}{K} \frac{(2KLM)^n}{n!} .$$